IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

M06/5/MATSD/SP2/ENG/TZ0/XX+

MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

Thursday 4 May 2006 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. *[Maximum mark: 17]*

(a) A function $f(x)$ is defined by $f(x) = 2x^2 - 10x + 60,\ -5 \le x \le 8$.

x	–5	0	2	5	8
$f(x)$	160	a	b	60	108

(i) Write down the values of a and b. *[2 marks]*

(ii) Using the values in the above table, draw the graph of $f(x)$ on a set of coordinate axes. Use a scale of 1 cm to represent 1 unit on the horizontal axis and 1 cm to represent 20 units on the vertical axis. *[4 marks]*

(iii) Show that the coordinates of the vertex of the graph are (2.5 , 47.5) *[3 marks]*

(iv) State the values of x for which the function is increasing. *[2 marks]*

(b) A second function $h(x)$ is defined by:

$$h(x) = 80,\ 0 \le x \le 8.$$

(i) On the same axes used for part (a), draw the graph of $h(x)$. *[2 marks]*

(ii) Find the coordinates of the point at which $f(x) = h(x)$. *[2 marks]*

(iii) Find the vertical distance from the vertex of the graph of $f(x)$ to the line $h(x)$. *[2 marks]*

2. *[Maximum mark: 19]*

(i) Let U be the set of all positive integers from 1 to 21 inclusive.

A, B and C are subsets of U such that:

A contains all the positive integers that are factors of 21,
B is the set of multiples of 7 contained in U,
C is the set of odd numbers contained in U.

(a) List all the members of set A. *[2 marks]*

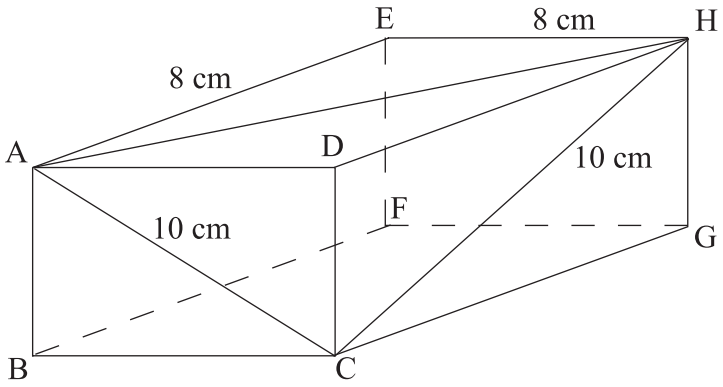
(b) Write down all the members of

(i) $A \cup B$,

(ii) $C' \cap B$. *[4 marks]*

(c) Find the probability that a member chosen at random from A is also a member of $A \cap B \cap C$. *[2 marks]*

(ii) The figure below shows a rectangular prism with some side lengths and diagonal lengths marked. $AC = 10\text{ cm}$, $CH = 10\text{ cm}$, $EH = 8\text{ cm}$, $AE = 8\text{ cm}$.



(not to scale)

(a) Calculate the length of AH. *[2 marks]*

(b) Find the size of angle ACH. *[3 marks]*

(c) Show that the total surface area of the rectangular prism is $320\,\text{cm}^2$. *[3 marks]*

(d) A triangular prism is enclosed within the planes ABCD, CGHD and ABGH. Calculate the volume of this prism. *[3 marks]*

3. *[Maximum mark: 17]*

The function $g(x)$ is defined by $g(x) = \frac{1}{8}x^4 + \frac{9}{4}x^2 - 5x + 7$, $x \geq 0$.

(a) Find $g(2)$. *[2 marks]*

(b) Calculate $g'(x)$. *[3 marks]*

The graph of the function $y = g(x)$ has a tangent T_1 at the point where $x = 2$.

(c) (i) Show that the gradient of T_1 is 8.

(ii) Find the equation of T_1. Write the equation in the form $y = mx + c$. *[5 marks]*

(d) The graph has another tangent T_2 at the point $\left(1, \frac{35}{8}\right)$. T_2 has zero gradient.

Write down the equation of T_2. *[2 marks]*

(e) (i) Sketch the graph of $y = g(x)$ in the region $0 \leq x \leq 3$, $0 \leq y \leq 22$.

(ii) Add the two tangents T_1 and T_2 to your sketch, in the correct positions. *[5 marks]*

4. *[Maximum mark: 19]*

(i) Alex invests 3600 euros in an account that pays a nominal rate of 5.4 % interest per year, compounding monthly. The interest is added to the account at the end of each month.

(a) Calculate the number of **whole** months it will take for Alex's investment to double. *[4 marks]*

(b) (i) Calculate the value of Alex's investment after nine years.

(ii) Find the rate of **simple interest** per year that would give the same value for the investment after nine years. *[6 marks]*

(ii) Annie is starting her first job. She will earn a salary of $ 26 000 in the first year and her salary will increase by 3 % every year.

(a) Calculate how much Annie will earn in her 5^{th} year of work. *[3 marks]*

Annie spends $ 24 800 of her earnings in her first year of work. For the next few years, inflation will cause Annie's living expenses to rise by 5 % per year.

(b) (i) Calculate the number of years it will be before Annie is spending more than she earns.

(ii) By how much will Annie's spending be greater than her earnings in that year? *[6 marks]*

5. *[Maximum mark: 18]*

(a) For his Mathematical Studies project, Marty set out to discover if stress was related to the amount of time that students spent travelling to or from school. The results of one of his surveys are shown in the table below.

Travel time (t mins) ↓	Number of students: high stress	moderate stress	low stress
$t \leq 15$	9	5	18
$15 < t \leq 30$	17	8	28
$30 < t$	18	6	7

He used a χ^2 test at the 5 % level of significance to find out if there was any relationship between student stress and travel time.

(i) Write down the null and alternative hypotheses for this test. *[2 marks]*

(ii) Write down the table of expected values. Give values to the nearest integer. *[3 marks]*

(iii) Show that there are 4 degrees of freedom. *[1 mark]*

(iv) Calculate the χ^2 statistic for this data. *[2 marks]*

The χ^2 critical value for 4 degrees of freedom at the 5 % level of significance is 9.488.

(v) What conclusion can Marty draw from this test? Give a reason for your answer. *[2 marks]*

(b) Marty asked some of his classmates to rate their level of stress out of 10, with 10 being very high. He also asked them to measure the number of minutes it took them to get from home to school. A random selection of his results is listed below.

Travel time (x)	13	24	22	18	36	16	14	20	6	12
Stress rating (y)	3	7	5	4	8	8	4	8	2	6

(i) Write down the value of the (linear) coefficient of correlation for this information. *[1 mark]*

(ii) Explain what a positive value for the coefficient of correlation indicates. *[1 mark]*

(iii) Write down the linear regression equation of y on x in the form $y = ax + b$. *[2 marks]*

(iv) Use your equation in part (iii) to determine the stress rating for a student who takes three quarters of an hour to travel to school. *[2 marks]*

(v) Can your answer in part (iv) be considered reliable? Give a reason for your answer. *[2 marks]*